Answer Keys:

Section-I

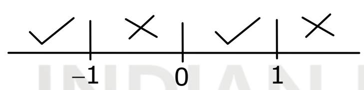
1	B	**2**	D	**3**	C	**4**	B	**5**	A	**6**	A	**7**	D
8	A	**9**	D	**10**	C	**11**	D	**12**	B	**13**	D	**14**	D
15	D	**16**	D	**17**	A	**18**	A	**19**	D	**20**	B		

Section-II

1	A	**2**	D	**3**	A	**4**	B	**5**	A	**6**	A	**7**	A
8	B	**9**	C	**10**	B	**11**	C	**12**	A	**13**	A	**14**	B
15	D	**16**	B	**17**	C	**18**	A	**19**	A	**20**	A	**21**	A
22	C	**23**	D	**24**	B	**25**	A	**26**	B	**27**	B	**28**	A
29	D	**30**	C										

Explanations:

Section-I

1. 

For $x > 1$, $(x-1)x(x+1) \le 0$ condition is not satisfied

For $0 \le x \le 1$, $(x-1)x(x+1) \le 0$ condition is satisfied

$-1 < x < 0$, $(x-1)x(x+1) \le 0$ condition is not satisfied

$x \le -1$, $(x-1)x(x+1) \le 0$ condition is satisfied

2. Since 70% of the employees received bonuses of at least 10,000, 30% of the employees received bonuses of less than 10,000. We know that 60 employees received bonuses of less than 10,000. If E is the number of employees, we can set up the following equation:
.30E = 60, E = 200
40% of the employees received bonuses of at least 50,000. Thus, (40% × 200) or 80 employees received bonuses of at least 50,000. 20% of the employees received bonuses of at least 1,00,000. Thus, (20% × 200) or 40 employees received bonuses of at least 1,00,000. If 80 employees received at least 50,000, and 40 employees received at least 1,00,000, then (80 - 40) or 40 employees received bonuses of at least 50,000 but less than 1,00,000.

3. $P\left(1+\frac{r}{100}\right)^5 = 3P, \quad P\left(1+\frac{r}{100}\right)^{10} = 9P$

4. Average = sum of terms / number of terms. In this question, we can apply the formula to the difference between the average we got and the average we were supposed to get:

$1.8 = \frac{\text{extra sum of terms}}{10} \Rightarrow 18 = \text{extra sum of terms}$

So, 'ut' is 18 more than 'tu'

'ut'= $10u+1\times t=10u+t$

'tu'=$10t+1\times u=10t+u$

'ut' - 'tu' = 9 (u-t)

i.e. 18 = 9 (u-t)

Therefore, u-t=2

5. Let Tap A fills 1 liter in a minute.
Then Tap B fills 2 liters per min (that is why Tap A is taking double time)
Together, they will fill 3 lt per min.
In 6 hours, they will fill 18liters (which is capacity of tank).
To fill 18liters (full tank), Tap A will take 18 hours

6. When there is a loss at 10% $\rightarrow$ 160 = 90% of CP_2

$\therefore C.P_2 = 177.37$

When there is a profit of 10% $\rightarrow$ 160 = 110% of CP_1

$\therefore C.P_1 = 145.45$

Total $C.P = 177.77 + 145.45 = 323.23$

$\text{Loss} = 3.23$

7. Let the ages of children is $x, (x+3), (x+6), (x+9) \& (x+12)$ yrs.

Then $x + (x+3) + (x+6) + (x+9) + (x+12) = 50$

$5x + 30 = 50 \Rightarrow x = 4$

8. $x = \frac{90}{360} \times 45{,}000 = 11{,}250\,\text{rs}; \quad y = \frac{120}{360} \times 45{,}000 = 15{,}000\,\text{rs}$

$z = \frac{150}{360} \times 45{,}000 = 18{,}750\,\text{rs};$

Hence in 1997 the costs are:

$x = 11{,}250 \times 1.1 = \text{Rs. } 12375$

$y = 15{,}000 \times 1.3 = \text{Rs. } 19500$

$z = 18{,}750 \times 1.2 = \text{Rs. } 22500$

$\text{Total cost} = 12375 + 19500 + 22500 = 54375$

9. $1-9 \quad 9\times 1\text{digits} = 9$

$10-99 \quad 90\times 2\text{digits} = 180$

$100-999 \quad 900\times 3\text{digits} = 2700$

$\overline{2889}$

2777th digit is of a 3 digit number

$2889-2777=112=37\times 3+1$

From 999, 37 numbers behind is 962. Its second digit is required answer. So answer is 6.

10. i. $\frac{\text{B in 2011}}{\text{C in 2012}}=\frac{15}{55}=\frac{3}{11}$

ii. Average $=\frac{10+15+40+40}{4}=\frac{105}{4}=26.25$

iii. Percentage increase in C = $\frac{30\text{-}15}{15}\times 100\% = 100\%$

Section-II: Technical

1. The two balls drawn may be both green, one green and one red or both red.

In these cases, the man receives 40paise, 30paise and 20paise respectively.

Let X be the amount the man receives. Then

$P[x=40]=P[\text{both green}]=\frac{^3C_2}{^5C_2}=0.3$

$P[x=30]=P[\text{one green one red}]=\frac{3C_1\times 2C_1}{5C_2}=0.6$

$P[x=20]=P[\text{both red}]=\frac{2C_2}{5C_2}=0.1$

$\therefore$ Probability distribution of x is

x	40	30	20		
P(x)	0.3	0.6	0.1		

$E(x)=\Sigma xP(x)=40\times 0.3+30\times 0.6+20\times 0.1=32\text{paise}$

2. After 1st clock pulse = 1101

After 2nd clock pulse = 1110

:

:

So on after 6th clock pulse =1011

3. $V_o = V\sin\omega t$

$\frac{dv_o}{dt} = V\omega\cos\omega t$

$\text{maximum output} = V\omega = 10\times10^6$

$V = \frac{10\times10^6}{2\times\pi\times2\times10^6} = 0.8V$

4. We will have

$Y = \overline{\overline{\overline{X}.\overline{Y}}.(1.0) + \overline{\overline{x.y}}.(1.0)}$

$= \overline{\overline{(x.y)(1.0)} + \overline{(x.y).(1.0)}}$

$= \overline{\overline{(x.y.1.0).(x.y.1.0)}} = x.y.1.0 = 0$

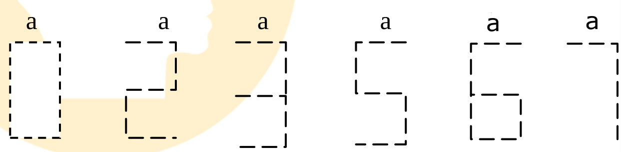
5. $\frac{\partial f}{\partial x} = nx^{n-1};\ \frac{\partial f}{\partial y} = ny^{n-1};\ \frac{\partial f}{\partial z} = nz^{n-1} \Rightarrow \nabla f = n\left[x^{n-1}i + y^{n-1}j + z^{n-1}k\right]$

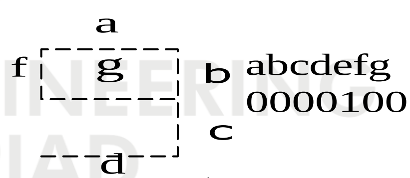
Since r = xi+yj+zk,

$\nabla f \cdot r = \left(nx^{n-1}x\right) + \left(ny^{n-1}y\right) + \left(nz^{n-1}z\right) = n\left(x^n + y^n + z^n\right) = nf$

7.

x	y	z	a
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

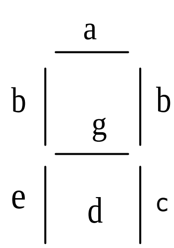




x \ yz	00	01	11	10
0	1		1	1
1		1	1	1

$a = y + xz + x'z'$

A seven-segment decoder with its segment display is shown below:
Segments outputs are given as a, b, c, d, e, f and g



8. $E_{Fn} = E_C - KT\ln\left(\frac{N_C}{N_D}\right)$

$$\Rightarrow \frac{E_C - E_{Fn2}}{E_C - E_{Fn1}} = \frac{KT_2\ln\left(\frac{NC}{ND}\right)}{KT_1\ln\left(\frac{NC}{ND}\right)}$$

$= E_C - E_{Fn} = 0.33ev$

New Fermi level lies 0.33eV below E_c. i.e. In n type material an increase in temperature move the Fermi level towards the midpoint of forbidden energy gap. Thus it moves downwards with reference to E_C.

9. $8V = 1600I_D + V_{DS} + 400I_D,$

$8 = 2000I_D + V_{DS};$

$2000I_D = 8 - V_{DS}$

$$I_D = \frac{8}{2000} - \frac{V_{DS}}{2000}$$

$$= \left(\frac{-1}{2000}\right)V_{DS} + \left(\frac{8}{2000}\right), \boxed{m = \frac{-1}{2000}};$$

$y = I_D; x = V_{DS}$

10. For an intrinsic semiconductor, the conductivity

$\sigma = n_i e(\mu_n + \mu_p)$

$(\mu_n + \mu_p) \propto T^{3/2}$ (Approximately)

$n_i = A_o t^{3/2} e^{\frac{-E_G}{2k_T}}$

$\therefore \quad \sigma = A_o(\mu_n + \mu_p)e^{-E_G/2kT}$

$A_o(\mu_n + \mu_e) = M.$ and

Resistivity $(\rho) = \frac{1}{\sigma} = \frac{1}{m}e^{E_G/2kT}$

Apply natural logarithm on bothsides,

$\therefore \quad \ln p = \ln\left(\frac{1}{m}\right) + \frac{E_G}{2kT}$

Thus, for the given semiconductors

Slope $= \frac{E_G}{2k}$

If forbidden energy gap E_G is more slope is more.

$\because$ The slope of B > A, B is more likely an insulator.

11. $\frac{60R_{min}}{2000+R_{min}} = 10 \Rightarrow R_{min} = 400\Omega$

$I_L = \frac{60-10}{2000} = 25\,mA$

$I_{L(min)} = 25\,mA - 20\,mA = 5\,mA$

$R_{L(max)} = \frac{10}{5m} = 2000\Omega$

$400\Omega < R < 2000\Omega$

12. $\underset{x\to\infty}{Lt}\left[\frac{x^2+5x+3}{x^2+x+2}\right]^x = \underset{x\to\infty}{Lt}\left(1+\frac{4x+1}{x^2+x+2}\right)^x = \underset{x\to\infty}{Lt}\left[\left(1+\frac{4x+1}{x^2+x+2}\right)^{\frac{x^2+x+2}{4x+1}}\right]^{\frac{x(4x+1)}{x^2+x+2}}$

$= e^{\underset{x\to\infty}{Lt}\frac{x(4x+1)}{x^2+4x+2}} = e^{\underset{x\to\infty}{Lt}\frac{4+\frac{1}{x}}{1+\frac{1}{x}+\frac{1}{x^2}}} = e^4$

13. $\in_r = 1 \to air$ $\qquad \frac{air \ \in_r = 1}{dielectric \ \in_r = 9}$

Reflected power $P_{ref} = \Gamma^2 P_{in}$

$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\frac{\eta_2}{\eta_1}-1}{\frac{\eta_2}{\eta_1}+1}$

$\frac{\eta_2}{\eta_1} = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = \frac{1}{3}$

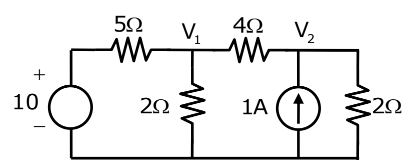
$\Gamma = \frac{\frac{1}{3}-1}{\frac{1}{3}+1} = \frac{-\frac{2}{3}}{\frac{4}{3}} = -\frac{1}{2}$

$P_{ref} = \left(\frac{1}{2}\right)^2 \times 10 = 2.5W$

14. Applying KCL at V_1 and V_2

$\frac{V_1-10}{5} + \frac{V_1}{2} + \frac{V_1-V_2}{4} = 0$ 1

$\frac{V_2-V_1}{4} + \frac{V_2}{2} = 1$ 2



By solving (1) and (2), we will get

$V_1 = 2.7V, \quad V_2 = 2.23V$

15. $z = \frac{10\angle 0}{5\angle -30} = 2\angle 30; 10\cos(\omega t + 30^o) = 10\sin(90 - \omega t - 30^o) = 10\sin(60 - \omega t)$

$= -10\sin(\omega t - 60) = -10\angle -60 V,$

$I = \frac{V}{Z} = \frac{-10\angle -60}{2\angle 30} = -5\angle -90 = 5\angle 90$

$= 5\sin(\omega t + 90) = 5\cos\omega t \text{ A}$

16. $\int_c Mdx + Ndy = \iint_R \left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right) dxdy, M = xy + y^2;\ N = x^2$

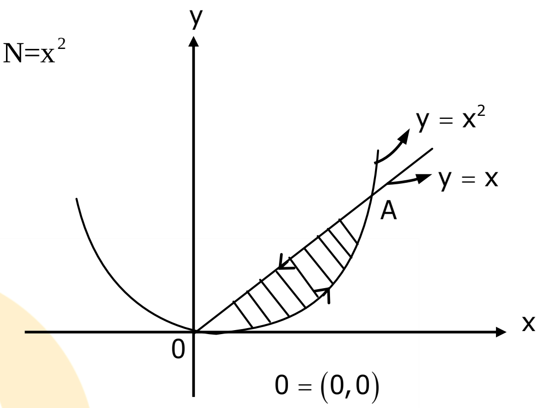
$\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y} = 2x - (x + 2y) = x - 2y$

$\iint_R \left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right) dxdy$

$= \int_{x=0}^{1} \int_{y=x^2}^{x} (x - 2y)\, dydx = \int_{x=0}^{1} \left[xy - y^2\right]_{y=x^2}^{x} dx$

$= \int_{x=0}^{1} \left[(x^2 - x^2) - (x^3 - x^4)\right] dx = \int_{x=0}^{1} (x^4 - x^3) dx$

$= \frac{1}{5} - \frac{1}{4} = \frac{-1}{20}$



17. (i) Linear, independent variable t, dependant variable y(t)

(ii) Non-linear as $y\frac{dy(t)}{dt}$ is a non-linear term.

(iii) Linear, the coefficients are all either constants or independent variable.

18. $\text{Tan}\,\theta = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

$\text{Tan}^2\,\theta = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\varepsilon_{r_2} \times \varepsilon_o}{\varepsilon_{r_1} \times \varepsilon_o}$

$\text{Tan}^2\,75^O = \frac{\varepsilon_{r_2} \times \varepsilon_o}{\varepsilon_{r_1} \times \varepsilon_o} \quad \varepsilon_{r_1} = 1$

$\Rightarrow \varepsilon_{r_2} = \text{Tan}^2\,75^O \Rightarrow \varepsilon_{r_2} = 13.93$

19. $H_i = \cos(10^8 t - \beta z)\ \hat{a}_y\ \ mA/m$

$$\frac{E_r}{E_i} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1/3$$

$$\frac{E_i}{H_i} = \eta \Rightarrow E_i = 120\pi(H_i)$$

$$E_r = \Gamma E_i = -40\pi\cos\left(10^8 t + \beta z\right)\hat{a}_x$$

20. We have, $e^{1/z} = 1 + \frac{1}{z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} .....$

$$\sin\frac{1}{z} = \frac{1}{z} - \frac{1}{3!z^3} + ...$$

There is an essential singularity at z = 0

The residue at z = 0 is coefficient of $\frac{1}{z}$ in Laurent series of integrand, which is 1

So $\oint_{|z|=1} e^{1/z} \sin\frac{1}{z} dz = 2\pi i$

21. $x[n] = \frac{2}{\pi}\frac{\text{Sin}\pi n}{\pi n}\ \ n \neq 0$; Energy of x[n] is

$$E = \sum_{n=-\infty}^{\infty}[x[n]]^2 = \left(\frac{2}{\pi}\right)^2 + \frac{2^2}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{\text{Sin}\pi n}{n}\right]^2$$

$$= \left(\frac{2}{\pi}\right)^2\left[1 + \frac{1}{\pi}\left\{\left(\frac{\sin\pi n}{1}\right)^2 + \left(\frac{\sin 2\pi}{2}\right)^2 + ...\right\}\right] = \left(\frac{2}{\pi}\right)^2$$

23. The $h(t) = \frac{\sin\omega_c(t - t_d)}{\pi(t - t_d)}$ So the option D is correct

24. $L_{eq} = 4 + 8 + 6 - (2\times 2) + (2\times 4) = 22H$

25. $\omega = 0,\ V_{out} = V_{in}$

$\omega = \infty,\ V_{out} = 0$

It represents low pass filter

26. $A_v = \frac{-h_{fe}R_c}{h_{ie} + (1 + h_{fe})R_e} = -20$

27.

Q(t)	K	G	Q(t+1)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

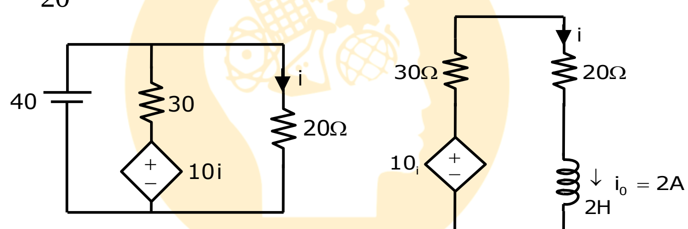
Q(t) \ KG	00	01	11	10
0			1	
1	1	1	1	

$Q(t+1) = KG + Qk'$

28. $X(z) = \left(\frac{0.5}{1-2z^{-1}}\right)z^{-1}$; ROC includes unit circle $\Rightarrow$ Left handed system

$X(n) = -0.5(2)^{n-1}u(-n);\ X(0) = -\frac{1}{4} = -0.25$

29. $t = 0^-$ $\quad \therefore i(0) = \frac{40}{20} = 2A$



at $t = 0^+$ the circuit becomes

$10\,i(t) = 30i(t) + 20i(t) + 2\frac{di}{dt}$

$2\frac{di}{dt} + 40\,i(t) = 0$

$i(t) = k\,e^{-20t}$

$i(t) = 2e^{-20t} \quad i(0) = 2A$

30. $V_0 = -\left(\frac{R}{R}\right) - \left(\frac{R}{R}\right)V_1 = V_1$