Answer Keys:

Section-I

1	D	**2**	D	**3**	B	**4**	C	**5**	B	**6**	C	**7**	B
8	C	**9**	B	**10**	C	**11**	B	**12**	C	**13**	A	**14**	D
15	B	**16**	B	**17**	A	**18**	C	**19**	A	**20**	C		

Section-II

1	C	**2**	B	**3**	A	**4**	A	**5**	D	**6**	A	**7**	A
8	D	**9**	B	**10**	B	**11**	A	**12**	C	**13**	B	**14**	A
15	A	**16**	A	**17**	B	**18**	A	**19**	D	**20**	A	**21**	C
22	B	**23**	B	**24**	A	**25**	C	**26**	C	**27**	D	**28**	D
29	A	**30**	A										

Explanations:

Section-I

1. We see that our RHS exponent is 11; therefore, we set our lowest exponent to 11.
$x-4$ is certainly smaller than $x-1$, so if we let $x-4=11$ we get:
$2^{14} - 2^{11} = (2^3)(2^{11}) - 2^{11} = 8(2^{11}) - 2^{11} = 7(2^{11})$

2. $l \xrightarrow{\uparrow 20\%} 1.2l$
$b \xrightarrow{\downarrow 10\%} 0.9b$
New perimeter = $2(1.2l + 0.9b)$
Increase in perimeter = $2(0.2l - 0.1b)$
Relation between l & b is not given
$\therefore$ We can't find out the percentage increase/decrease

3. If the average weight of the entire group was twice as close to the average weight of the men as it was to the average weight of the women, there must be twice as many men as women. With a 2:1 ratio of men to women of, 33 1/3% (i.e. 1/3) of the group must have been women. Consider the following rule and its proof.

Alligation rule: The ratio that determines how to weight the averages of two or more subgroups in a weighted average also reflects the ratio of the distances from the weighted average to each subgroup's average.

4. Let x be the speed of stairway
$25 + 15x = 13 + 24x$
Or $x = \frac{4}{3}$
$\therefore 25 + 15 \times \frac{4}{3} =$ 45 steps are in total

5. Rate and time are always inversely related:
AB's rate is 6/5, so in 1 hour, AB will do 5/6 of the job.
AC's rate is 3/2, so in 1 hour, AC will do 2/3 of the job.
BC's rate is 2, so in 1 hour, BC will do 1/2 of the job.

Let the number of units in the total job be a number that is a multiple of 6, 3, and 2; let's say there are 18 units in the total job.
Then, in one hour:
AB will complete 5/6*18 = 15 units;
AC will complete 2/3*18 = 12 units;
BC will complete 1/2*18 = 9 units.

Summing up:
2 As, 2 Bs, and 2Cs complete 36 units.
So, in one hour, 2 of each of the pumps will complete two jobs. Therefore, it will take 1 of each of the pumps 1 hour to complete the job.

6. It may be helpful to put the question in algebraic terms.
The tip will be equal to a constant, c, plus an amount that is proportional to the bill: kb, where k is the fraction of the bill, and b is the amount of the bill.
So the tip will be $c+kb$, and since we know the bill for the meal is 600/-, the tip will be $c+600k$.
$60 = c+700k$
$40 = c+450k$
Subtract the equations, giving you the result:
$20 = 250k \Rightarrow k = 0.08$
Then plug k back into one of the equations:
$60 = c+700(0.08) \Rightarrow c = 4$
Therefore, tip = 4+600(0.08) = 52

7. 4 men can go in five hotels in 5^4 ways.
Number of ways in which 4 men can go into different hotel

$$= 5_{P4} = \frac{5!}{(5-4)!} = 5!$$

$$\therefore \text{ Required proability} = \frac{5!}{5^4} = \frac{120}{625} = \frac{24}{125}$$

8. $|A \cup B| = 40$
$|A \cup B| = |A| + |B| - |A \cap B|$
$40 = |A| + 22 - 12$
$|A| = 30$
A=30 enrolled for English & included both subjects
Number of students enrolled for English only = 30-12=18.

9. Let Mr. Vikas buys LCM (8, 5, 9) = 360 Apples of each variety.

Amount spent on the 1st variety = $\frac{360}{8}$ = 45 rs.

Amount spent on the 2nd variety = $\frac{360}{5}$ = 72 rs.

∴ Total amount spent = 45+72 = Rs.117

Now the total (360+360) = 720 Apples are sold at 9 per rupee

∴ Total revenue = $\frac{720}{9} = 80$

Hence the loss = 117-80 = 37

$\therefore\ \text{Loss}\% = \frac{37}{117}\times 100 = 31.62\%$

10. Total respondents in the 21 – 30 age group = 33.

Out of them 33 – 12 = 21 like any program other than singing/dancing

$\therefore \% = \left(\frac{21}{33}\right)\times 100 = 63.63 \approx 64\%$

15. Junior/ Senior/ prior are followed by "to"

Section-II

1. Expectation of $X = E(x) = \text{Average} = \text{mean} = \frac{1}{n}\sum_{1}^{n} X_i = \sum_{i=1}^{n} P_i x_i$

Where P_i is probability of occurrence of x_i

Let $q = 1 - p$

Then probability of success in 1st, 2nd and 3rd trials are p, qp, q^2p

$\therefore E(x) = 0\cdot p + 1\cdot qp + 2\cdot q^2p + .. = qp\left[1 + 2q + 3q^2 + ... + nq^{n+1} + ...\right]$

$= \frac{qp}{(1-q)^2} = \frac{qp}{p^2} = \frac{q}{p}$

3. For Si variation in temperature /°C is -2.5 mV

i.e, $\frac{dv}{dT} = -2.5\ \text{mV}/^{\circ}\text{C}$

At T_1= 300 k or 23°C, the forward bias voltage V_1=0.6 V

At T_2= 310 k or 33°C, the forward bias voltage is V_2.

To maintain a constant current, $\frac{V_2 - V_1}{T_2 - T_1} = -2.5\times 10^{-3}$

$\Rightarrow \frac{V_2 - 0.6}{33 - 23} = -2.5\times 10^{-3}$

$\Rightarrow V_2 = 0.6 - \left(2.5\times 10^{-2}\right)$

$\Rightarrow \boxed{V_2 = 0.575\text{V}}$

4. $G(S)=\dfrac{k}{S(S+T)}$ $\therefore$ K is negative

$$\angle G(j\omega)=180-90-\mathrm{Tan}^{-1}\left(\frac{\omega}{T}\right)=90^0-\mathrm{Tan}^{-1}\left(\frac{\omega}{T}\right)$$

At $\omega=0\Rightarrow\angle G(j\omega)=90^0$ and $\omega=\infty\Rightarrow\angle G(j\omega)=0^0$

5. $$I=\iint_R xy(x+y)\,dydx=\int_{x=0}^{1}\int_{y=x^2}^{x}\left(x^2y+xy^2\right)dydx$$

$$=\int_{x=0}^{1}\left\{x^2\left(\frac{y^2}{2}\right)_{y=x^2}^{x}+x\left[\frac{y^3}{3}\right]_{y=x^2}^{x}\right\}dx$$

$$=\int_{x=0}^{1}\left[\frac{x^4}{2}-\frac{x^6}{2}+\frac{x^4}{3}-\frac{x^7}{3}\right]dx=\frac{x^5}{10}-\frac{x^7}{14}+\frac{x^5}{15}-\frac{x^8}{24}\Bigg|_{x=0}^{1}=\frac{3}{56}$$

6. $V(t)=t$ for $0<t\le 1s$

$$i(t)=c\frac{dv(t)}{dt}=2\times\frac{dv(t)}{dt}=2\times 0.8=1.6A$$

7. $P_{rad}=I_{rms}^2.R_{rad}$ or $R_{rad}=\dfrac{4}{36}=0.11\Omega$

$$R_{rad}=80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

$$\Rightarrow\sqrt{\frac{0.11}{80\pi^2}}=\frac{dl}{\dfrac{3\times10^8}{250\times10^6}}$$ or $dl=0.014m=1.42cm$

8. Controllable:

$$A=\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix},\quad B=\begin{bmatrix}1\\0\end{bmatrix},\quad C=\begin{bmatrix}0 & 1\end{bmatrix}$$

$$AB=\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix},\ U=[B:AB]=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix},\ |U|=1-0=1\neq0$$

Observability:

$$V=\left[C^T;A^TC^T\right],C^T=\begin{bmatrix}0\\1\end{bmatrix},$$

$$A^TC^T=\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

$$\therefore V=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$$\therefore |V|=0-1=-1\neq0$$

$\therefore$ System is controllable & observable

9. $$\frac{10-V_1}{5}+\frac{V_0-V_1}{5}=0 \Rightarrow 10=2V_1-V_0 \quad ....(1)$$

$$\frac{V_1}{10}+I_L+\frac{V_1-V_0}{10}=0 \Rightarrow I_L=\frac{-2V_1+V_0}{10} \Rightarrow I_L=\frac{-(10)}{10}=-1A$$

10. $$x_P(\Omega)=\frac{2e^{-j\Omega}}{1-0.25e^{-j2\Omega}}$$

$$=\frac{2e^{-j\Omega}}{\left(1-0.5e^{-J\Omega}\right)\left(1+0.5e^{-j\Omega}\right)}$$

$$x_P(\Omega)=\frac{2}{1-0.5e^{-j\Omega}}-\frac{2}{1+0.5e^{-j\Omega}}$$

$$x_P(\Omega)\xleftrightarrow[\text{pairs}]{\text{DTFT}}x[n]=2(0.5)^n u(n)-2(-0.5)^n u(n)$$

11. 0000 0000 0000 1111

$\Downarrow$

FFFF FFFF FFFF FFFF

12. $$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)};$$

$$x_n-\frac{\left(3x_n^{\,2}+2x_n+1\right)}{6x_n+2}=\frac{3x_n^{\,2}-1}{6x_n+2}$$

13. As there are no independent sources the Thevenin's and Norton equivalent will have 0V and 0A sources. To find R_{TH}, a 1A source is connected as $R_{TH}=\frac{V_x}{1}$

Writing a nodal equation at n,

$$1-\frac{V_x}{100}=\frac{V_x}{3000}+\frac{V_x-1000\,i_x}{100}$$

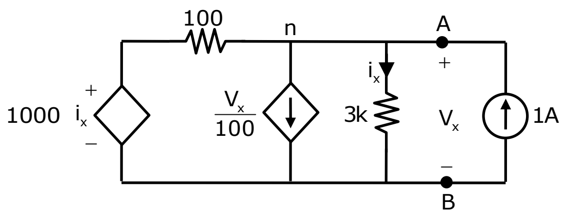
$$\Rightarrow 1-\frac{V_x}{100}=\frac{V_x}{3000}+\frac{V_x-1000\frac{V_x}{3000}}{100} \qquad \left(\therefore i_x=\frac{V_x}{3000}\right)$$

$$\Rightarrow 1-\frac{V_x}{100}=\frac{V_x}{3000}+\frac{2}{3}\frac{V_x}{100}$$

$$\Rightarrow 0.01V_x+0.0003V_x+0.0067V_x=1$$

$$\Rightarrow V_x=58.82\,V$$

$$\therefore R_{TH}=\frac{V_x}{I}=58.82\Omega$$



14. $I_h = AJ_h = -AqD_p.\frac{dp}{dx}$

$D_p = \mu_p.V_T = 640 \times 0.025 = 16\ cm^2/s$

$I_h = -10^{-4} \times 1.6 \times 10^{-19} \times 16 \times \left[\frac{-5\times 10^{14}}{2\times 10^{-4}}\right] = 6.4\times 10^{-4}\ A = 640\ \mu A$

15. Since the rate of transmission is $R = 10^5$ bits / second, the bit interval T_b is 10^{-5}s.

The probability of error in a binary PAM system is $P(e) = Q\left[\sqrt{\frac{2E_b}{N_0}}\right]$

where $E_b = A^2T_b$ with $P(e) = P_2 = 10^{-6}$

$\Rightarrow \sqrt{\frac{2E_b}{N_0}} = 4.75 \Rightarrow E_b = \frac{4.75^2 N_0}{2} = 0.112813$

Thus $A^2T_b = 0.112813 \Rightarrow A = \sqrt{0.112813\times 10^5} = 106.21$

16. $\int_0^\infty\int_y^\infty xe^{-\frac{x}{y^2}}dxdy = \int_{y=0}^{\infty}\left\{\int_{x=y} xe^{-x^2/y}dx\right\}dy$

$= \int_{y=0}^{\infty}\left\{\int_{x=y}^{\infty}\left[\frac{-y}{2}e^{\frac{-x^2}{y}}\left(\frac{-2x}{y}dx\right)\right]\right\}dy = \int_0^\infty\left\{\frac{-y}{2}\left[e^{\frac{-x^2}{y}}\right]_{x=y}^{\infty}\right\}dy = \int_0^\infty \frac{y}{2}e^{-y}dy$

$= \frac{1}{2}\left\{\left[\frac{ye^{-y}}{-1}\right]_0^\infty - \int_0^\infty\left(\frac{e^{-y}}{-1}\right)dy\right\} = \frac{1}{2}\left[\frac{e^{-y}}{-1}\right]_0^\infty = \frac{1}{2} = 0.5$

17. DFT $X(k) = \sum_{n=0}^{N-1} x(n)W_6^{kn}$

for N=6; $X(k) = 4 + 3W_6^k + 2W_6^{2k} + W_6^{3k}$

$Y[k] = W_6^{4k}X(k) = 4W_6^{4k} + 3W_6^{5k} + 2W_6^{6k} + W_6^{7k}$

Using periodicity property, $W_6^{6k} = 1;\ W_6^{7k} = W_6^k$

$Y[k] = 4W_6^{4k} + 3W_6^{5k} + 2 + W_6^{1k}$

$\because Y[k] = \sum_{n=0}^{N-1} y(n)W_6^{kn}$

$y(4) = 4,\ y(5) = 3,\ y(0) = 2,\ y(1) = 1$ and $y(2) = y(3) = 0$

$\therefore y(n) = \{\underset{\uparrow}{2}, 1, 0, 0, 4, 3\}$

18. 8 bit Johnson counter will divide frequency by 16 times
4 bit parallel counter will divide frequency by 16 times
8 bit ring counter will divide frequency by 8 times
Input frequency = 16 × 6 × 8 × 10 = 20480

19. At resonance $\left[V_c = Q_o V_s\right]$

20. $dy = \left(\frac{2}{x} - 4x^2\right)dx$

$\Rightarrow y = 2\ln x - \frac{4x^3}{3} + c$

22. $\nabla.\bar{D} = P$ $\quad J_D = \frac{\partial D}{\partial t}$

$\nabla.\bar{B} = 0$ $\quad \bar{B} = \mu\bar{H}$

$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$ $\quad H = \frac{\bar{B}}{\mu}$ $\quad D = \varepsilon_0 \bar{E}$

$\nabla \times \bar{H} = J_C + J_D;$ $\quad J_D = 0 \quad -\frac{\partial B}{\partial t} = 0$

23. We are given $y(t) = \int_{t-T}^{t} x(T)dT$

For x (t) = δ(t), the impulse response of this running integrator is

$h(t) = \int_{t-T}^{t} \delta(\tau)d\tau = 1 \text{ for } t - T \le 0 \le t(\text{ by definition,})$

or, equivalently, $0 \le t \le T$

Correspondingly, the frequency response of the running integrator is

$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt = \int_{0}^{T} e^{-j2\pi ft}dt$

$= \frac{1}{j2\pi ft}\left[1 - \exp(-j2\pi fT)\right] = T\sin c(fT)\exp(-j\pi fT)$

Hence, the PSD $S_Y(f)$ is defined in terms of the PSD $S_X(f)$ as follows

$S_Y(f) = |H(f)|^2 S_x(f)$

24. Upper cross over voltage when, v_o = +10v, At upper threshold point

$\left(\frac{5}{5+20}\right)10 = \left(\frac{10}{10+20}\right)2 + \left(\frac{20}{10+20}\right)V_{th}$

$\Rightarrow V_{Th} = 2V$

Lower cross over voltage when v_o = -10V

$\left(\frac{5}{5+20}\right)(-10) = \left(\frac{10}{10+20}\right)(2) + \left(\frac{20}{10+20}\right)V_{TL}$

$\Rightarrow V_{TL}$ = -4V

26. $V_1 = Av_2 + B(-I_2)$

$I_1 = CV_2 + D(-I_2)$

Making $V_2 = 0$

$I_1 = -I_2$

or $\left[D = \frac{I_1}{-I_2} = 1\right]$

Making $I_2 = 0$

$V_2 = 4(I_1 - 0.1V_2)$

$V_2 = 4I_1 - 0.4V_2$

$1.4V_2 = 4I_1$

$V_1 - 5I_1 + 0.3V_1 = 0$

$1.3V_t = 5I_1 = -5I_2$

$\left[\frac{V_1}{-I_2} = B = \frac{5}{1.3} = 3.846\right]$

or $\left[C = \frac{I_1}{V_2} = \frac{1.4}{4} = 0.35\right]$

$V_1 - 5I_1 + 0.3V_1 - V_2 = 0$

$1.3V_1 - 5 \times 0.35V_2 - V_2 = 0$

$1.3V_1 = 2.75V_2$

or $\left[A = \frac{V_1}{V_2} = 2.11\right]$

or $\begin{bmatrix} 2.11 & 3.846 \\ 0.33 & 1 \end{bmatrix}$

27. $\Delta\theta = k\int_{-\infty}^{t} m(\tau)d\tau;\ \Delta\omega = \frac{d}{dt}\Delta\theta = km(t) \Rightarrow \Delta f = \frac{k}{2\pi}m(t)$, For given $f_M(m)$

mean=0,S.D.=2; $\Delta f_{rms} = \frac{k}{2\pi} \times 2 = \frac{k}{\pi}$

28. First multiplier output=10cos4000 πt

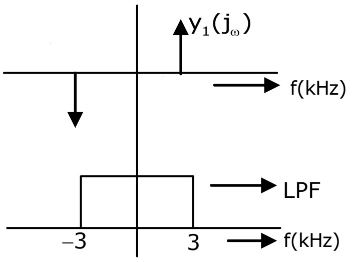
$\cos 4000\pi t$ is shifted by $90^o \Rightarrow \cos(4000\pi \pm 90)t = \sin 4000\pi t$

$\therefore$ second multiplier output $= 10\cos 4000\pi t \sin 4000\pi t$

$y_1(t) = 5\sin 8000\pi t$

$y_1(i\omega) = \frac{5\pi}{j}\left[\delta(\omega - 8000\pi) - \delta(\omega + 8000\pi)\right]$

y_1(t) is passed through an LPF of cut off frequency 3kHz.



On multiplying above two signals the output y(t) is zero.

29. For the 1st stage alone,

$$A_V = \frac{-\beta_0 R_C}{R_S + r_\pi}; \quad r_\pi = \frac{\beta_0}{g_m} = 125 \times 25 = 3.125 k\Omega;$$

$$A_{V_1} = -\frac{125 \times 1.2}{0.6 + 3.125} = -40.3$$

For the 2nd stage, $R_S = R_{C_1} = 1.2k$ & r_π is same $R_{C_2} = 1.2k$

$$\therefore A_{V_2} = \frac{-125 \times 1.2}{1.2 + 3.125} = -34.7$$

$\therefore$ Overall voltage gain $A_V = A_{V_1} \times A_{V_2} = 1400$

30. Applying Routh Hurwitz $s^5 - s^4 - s^3 - 17s^2 - 90s - 72$

s^5	1	-1	-90
s^4	-1	-17	-72
s^3	$\frac{1-(-17)}{-1} = -18$	$\frac{90+72}{-1} = -162$	0
s^2	-8	-72	
s^1	$0/-16$	0	
s^0	-72		

As auxiliary equation is $-8s^2 - 72 = 0$
Differentiating we get $-16s = 0$
Five roots in total, 2 on imaginary axis, one sign change on RHS, hence
$LHS = 5 - (2+1) = 2$.