Answer Keys:

Section-I

1	D	**2**	D	**3**	B	**4**	C	**5**	B	**6**	C	**7**	B
8	C	**9**	B	**10**	C	**11**	B	**12**	C	**13**	A	**14**	D
15	B	**16**	B	**17**	A	**18**	C	**19**	A	**20**	C		

Section-II

1	C	**2**	C	**3**	A	**4**	A	**5**	D	**6**	D	**7**	D
8	D	**9**	B	**10**	A	**11**	A	**12**	C	**13**	B	**14**	C
15	C	**16**	A	**17**	B	**18**	A	**19**	A	**20**	A	**21**	C
22	A	**23**	D	**24**	A	**25**	D	**26**	C	**27**	B	**28**	D
29	A	**30**	D										

Explanations:

Section-I

1. We see that our RHS exponent is 11; therefore, we set our lowest exponent to 11.
$x-4$ is certainly smaller than $x-1$, so if we let $x-4=11$ we get:
$2^{14} - 2^{11} = (2^3)(2^{11}) - 2^{11} = 8(2^{11}) - 2^{11} = 7(2^{11})$

2. $l \xrightarrow{\uparrow 20\%} 1.2l$
$b \xrightarrow{\downarrow 10\%} 0.9b$
New perimeter = $2(1.2l + 0.9b)$
Increase in perimeter = $2(0.2l - 0.1b)$
Relation between l & b is not given
$\therefore$ We can't find out the percentage increase/decrease

3. If the average weight of the entire group was twice as close to the average weight of the men as it was to the average weight of the women, there must be twice as many men as women. With a 2:1 ratio of men to women of, 33 1/3% (i.e. 1/3) of the group must have been women. Consider the following rule and its proof.

Alligation rule: The ratio that determines how to weight the averages of two or more subgroups in a weighted average also reflects the ratio of the distances from the weighted average to each subgroup's average.

4. Let x be the speed of stairway
$25 + 15x = 13 + 24x$
Or $x = \frac{4}{3}$
$\therefore 25 + 15 \times \frac{4}{3} = 45$ steps are in total

5. Rate and time are always inversely related:
AB's rate is 6/5, so in 1 hour, AB will do 5/6 of the job.
AC's rate is 3/2, so in 1 hour, AC will do 2/3 of the job.
BC's rate is 2, so in 1 hour, BC will do 1/2 of the job.

Let the number of units in the total job be a number that is a multiple of 6, 3, and 2; let's say there are 18 units in the total job.
Then, in one hour:
AB will complete 5/6*18 = 15 units;
AC will complete 2/3*18 = 12 units;
BC will complete 1/2*18 = 9 units.

Summing up:
2 As, 2 Bs, and 2Cs complete 36 units.
So, in one hour, 2 of each of the pumps will complete two jobs. Therefore, it will take 1 of each of the pumps 1 hour to complete the job.

6. It may be helpful to put the question in algebraic terms.
The tip will be equal to a constant, c, plus an amount that is proportional to the bill: kb, where k is the fraction of the bill, and b is the amount of the bill.
So the tip will be $c+kb$, and since we know the bill for the meal is 600/-, the tip will be $c+600k$.

$60 = c+700k$

$40 = c+450k$

Subtract the equations, giving you the result:

$20 = 250k \Rightarrow k = 0.08$

Then plug k back into one of the equations:

$60 = c+700(0.08) \Rightarrow c = 4$

Therefore, tip = 4+600(0.08) = 52

7. 4 men can go in five hotels in 5^4 ways.
Number of ways in which 4 men can go into different hotel

$$= 5_{P4} = \frac{5!}{(5-4)!} = 5!$$

$$\therefore \text{Required proability} = \frac{5!}{5^4} = \frac{120}{625} = \frac{24}{125}$$

8. $|A \cup B| = 40$

$|A \cup B| = |A| + |B| - |A \cap B|$

$40 = |A| + 22 - 12$

$|A| = 30$

A=30 enrolled for English & included both subjects
Number of students enrolled for English only = 30-12=18.

9. Let Mr. Vikas buys LCM (8, 5, 9) = 360 Apples of each variety.

Amount spent on the 1st variety = $\frac{360}{8} = 45$ rs.

Amount spent on the 2nd variety = $\frac{360}{5} = 72$ rs.

∴ Total amount spent = 45+72 = Rs.117

Now the total (360+360) = 720 Apples are sold at 9 per rupee

∴ Total revenue = $\frac{720}{9} = 80$

Hence the loss = 117-80 = 37

$\therefore \text{ Loss\%} = \frac{37}{117} \times 100 = 31.62\%$

10. Total respondents in the 21 – 30 age group = 33.

Out of them 33 – 12 = 21 like any program other than singing/dancing

$\therefore \% = \left(\frac{21}{33}\right) \times 100 = 63.63 \approx 64\%$

15. Junior/ Senior/ prior are followed by "to"

Section-II

1. Expectation of $X = E(x) = \text{Average} = \text{mean} = \frac{1}{n}\sum_{1}^{n} X_i = \sum_{i=1}^{n} P_i x_i$

Where P_i is probability of occurrence of x_i

Let $q = 1 - p$

Then probability of success in 1st, 2nd and 3rd trials are p, qp, q^2p

$$\therefore E(x) = 0 \cdot p + 1 \cdot qp + 2 \cdot q^2 p + .. = qp\left[1 + 2q + 3q^2 + ... + nq^{n+1} + ...\right]$$

$$= \frac{qp}{(1-q)^2} = \frac{qp}{p^2} = \frac{q}{p}$$

2. $\text{kVA} = \frac{992}{0.8} = 1240 \text{ kVA}$

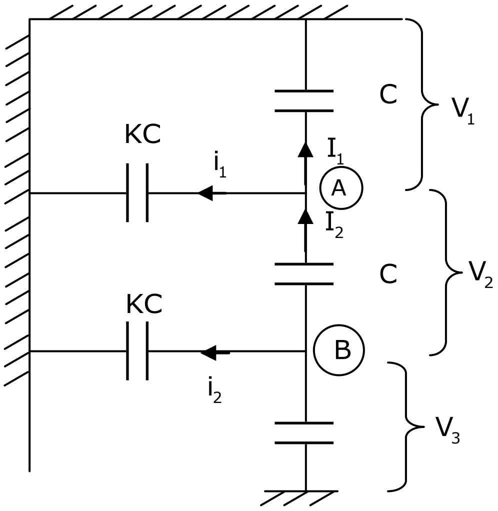
$$\text{Current } I = \frac{1240}{\sqrt{3} \times 6.6} = 108.47 \text{ A}$$

$$\text{Phase Voltage} = \frac{6.6 \times 10^3}{\sqrt{3}} = 3810.51\text{V}$$

$$E_{ph} = \sqrt{(V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi - I_a X_s)^2} = 3212.28\text{V}$$

$$\therefore \text{Regulation} = \frac{3212.28 - 3810.5}{3810.5} = -15.7\%$$

3. At A,



$$K = \frac{C_1}{C} = \frac{1}{6}$$

$$I_2 = i_1 + I_1$$

$$V_2 \omega c = V_1 \omega_c + V_1 \omega (kc)$$

$$V_2 = V_1 + V_1 k = V_1.k = V_1(1+k)$$

$$\boxed{V_2 = V_1(1+k)}$$

At B $i_3 = i_2 + I_2$

$$V_3 \omega c = (V_1 + V_2) kc\omega + V_2 \omega c$$

$$V_3 = (V_1 + V_2)k + V_2$$

$$\boxed{V_3 = V_1.k + V_2(1+k)}$$

$$V_2 = V_1\left(1+\frac{1}{6}\right) = \frac{7}{6} v_1 = 1.16\ V_1$$

$$V_3 = V_1\left(\frac{1}{6}\right) + V_2\left(1+\frac{1}{6}\right) = \frac{V_1}{6} + \frac{7}{6}\frac{7}{6}.v_1 = \frac{V_1}{6} + \frac{49}{36} V_1 = v_1\left[\frac{1}{6} + \frac{49}{36}\right]$$

$$V_3 = V_1\left[\frac{55}{36}\right]$$

Given $V_3 = 20kv$

$$\therefore 20 = V_1\left(\frac{55}{36}\right) \Rightarrow V_1 = 13.09\ KV$$

$$V_2 = V_1(1+k) = 13.09\left(1+\frac{1}{6}\right)$$

$$V_3 = 20kv$$

Total voltage across string $= V_1 + V_2 + V_3 = 48.36KV$

4. $G(S) = \dfrac{k}{S(S+T)}$ $\therefore$ K is negative

$$\angle G(j\omega) = 180 - 90 - Tan^{-1}\left(\frac{\omega}{T}\right) = 90^0 - Tan^{-1}\left(\frac{\omega}{T}\right)$$

At $\omega = 0 \Rightarrow \angle G(j\omega) = 90^0$ and $\omega = \infty \Rightarrow \angle G(j\omega) = 0^0$

5. $$I = \iint_R xy(x+y)dydx = \int_{x=0}^{1}\int_{y=x^2}^{x} (x^2y + xy^2)dydx$$

$$= \int_{x=0}^{1}\left\{x^2\left(\frac{y^2}{2}\right)_{y=x^2}^{x} + x\left[\frac{y^3}{3}\right]_{y=x^2}^{x}\right\}dx$$

$$= \int_{x=0}^{1}\left[\frac{x^4}{2} - \frac{x^6}{2} + \frac{x^4}{3} - \frac{x^7}{3}\right]dx = \frac{x^5}{10} - \frac{x^7}{14} + \frac{x^5}{15} - \frac{x^8}{24}\Bigg|_{x=0}^{1} = \frac{3}{56}$$

6. $V(t) = t$ for $0 < t \le 1s$

$$i(t) = c\frac{dv(t)}{dt} = 2\times\frac{dv(t)}{dt} = 2\times 0.8 = 1.6A$$

7. Time period $T = t_{on} + t_{off} = (1+1.5) = 2.5msec$,

Duty cycle $\alpha = \frac{T_{on}}{T} = \frac{1}{2.5} = 0.4$

$$\text{Form-factor (FF)} = \frac{\text{RMS Value}}{\text{Average Value}} = \frac{\sqrt{\alpha}.E_{dc}}{\alpha.E_{dc}}$$

$$= \frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{0.4}} = 1.58$$

$$\text{Ripple factor (RF)} = \sqrt{(FF)^2 - 1} = \sqrt{\frac{1}{\alpha} - 1}$$

$$= \sqrt{\frac{1-\alpha}{\alpha}} = \sqrt{\frac{1-0.4}{0.4}} = 1.23$$

8. Controllable:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$U = [B : AB] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad |U| = 1 - 0 = 1 \ne 0$$

Observability:

$$V = \left[C^T ; A^T C^T\right], C^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$A^T C^T = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\therefore V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \therefore \; |V| = 0 - 1 = -1 \ne 0$$

$\therefore$ System is controllable & observable

9. $$\frac{10 - V_1}{5} + \frac{V_0 - V_1}{5} = 0$$

$$\Rightarrow 10 = 2V_1 - V_0 \quad \text{.......(1)}$$

$$\frac{V_1}{10} + I_L + \frac{V_1 - V_0}{10} = 0$$

$$\Rightarrow I_L = \frac{-2V_1 + V_0}{10} \Rightarrow I_L = \frac{-(10)}{10} = -1A$$

10. The turn on loss $= \frac{1}{6} \times v_{dc} \times I_o \times T_{on}$

$\therefore$ Turn on energy loss $= \frac{1}{6} \times v_{dc} \times I_o \times T_{on}$

Turnoff energy loss $= \frac{1}{6} \times v_{dc} \times I_o \times T_{off}$

$\therefore$ Swtching loss $= \frac{1}{6} \times 100 \times 20 \times 0.1 \times 10^{-6} + \frac{1}{6} \times 100 \times 20 \times 0.1 \times 10^{-6} = 66.67 \times 10^{-6}\,J$

conduction energy loss $= \left(v_o \times I_o + (I_o)^2 \times R_d\right) \times 0.1 \times 10^{-3} = [1x20 + (20)^2 xR_d]x10^{-4}$

$$\therefore \text{Avergage power loss} = \frac{\left(66.67 \times 10^{-6} + [20 + 400R_d] \times 10^{-4}\right)}{0.2 \times 10^{-3}} = \frac{\Delta T}{\theta_{JC}}$$

$\Rightarrow 66.674 \times 10^{-6} + 20 \times 10^{-4} + 400R_d \times 10^{-4} = 100 \times 0.2 \times 10^{-3} \Rightarrow R_d = 0.448\Omega$

11. 0000 0000 0000 1111

$\Downarrow$

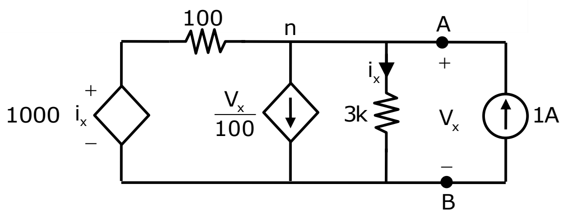
FFFF FFFF FFFF FFFF

12. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$;

$$x_n - \frac{(3x_n^{\ 2} + 2x_n + 1)}{6x_n + 2} = \frac{3x_n^{\ 2} - 1}{6x_n + 2}$$

13. As there are no independent sources the Thevenin's and Norton equivalent will have 0V and 0A sources. To find R_{TH}, a 1A source is connected as $R_{TH} = \frac{V_x}{1}$

Writing a nodal equation at n,



$$1 - \frac{V_x}{100} = \frac{V_x}{3000} + \frac{V_x - 1000\,i_x}{100}$$

$$\Rightarrow 1 - \frac{V_x}{100} = \frac{V_x}{3000} + \frac{V_x - 1000\frac{V_x}{3000}}{100}$$

$$\left(\therefore i_x = \frac{V_x}{3000}\right)$$

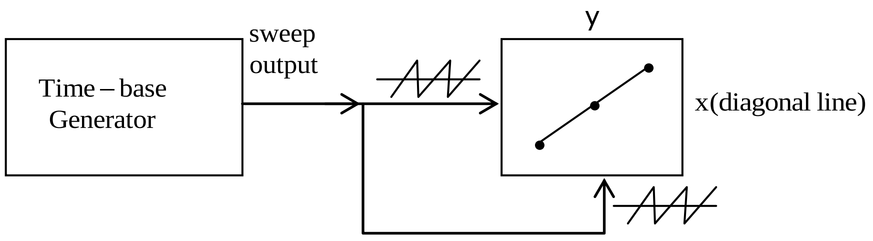
$$\Rightarrow 1 - \frac{V_x}{100} = \frac{V_x}{3000} + \frac{2}{3}\frac{V_x}{100}$$

$\Rightarrow 0.01V_x + 0.0003V_x + 0.0067\,V_x = 1$

$\Rightarrow V_x = 58.82\,V$

$$\therefore R_{TH} = \frac{V_x}{I} = 58.82\Omega$$

14.



15. $\alpha = 30^\circ$

$$V_0 = \frac{3 v_{ms}}{\pi}\cos\alpha = \frac{3\sqrt{2}.440}{\pi}\cos 30 = 514.59\text{V}$$

$$V_0 = E + I_O R$$

$$\therefore I_0 = \frac{V_0 - E}{R} = \frac{514.59 - 300}{20} = 10.72\text{A}$$

$$\text{Power delivered to load} = V_0 I_0 = (514.59)(10.72) = 5521.54\,\text{W}$$

16. $$\int_0^\infty\int_y^\infty x e^{-\frac{x}{y^2}} dxdy = \int_{y=0}^{\infty}\left\{\int_{x=y}^{\infty} x e^{-x^2/y} dx\right\} dy$$

$$= \int_{y=0}^{\infty}\left\{\int_{x=y}^{\infty}\left[\frac{-y}{2} e^{\frac{-x^2}{y}}\left(\frac{-2x}{y}dx\right)\right]\right\}dy = \int_0^\infty\left\{\frac{-y}{2}\left[e^{\frac{-x^2}{y}}\right]_{x=y}^{\infty}\right\}dy = \int_0^\infty \frac{y}{2} e^{-y} dy$$

$$= \frac{1}{2}\left\{\left[\frac{ye^{-y}}{-1}\right]_0^\infty - \int_0^\infty\left(\frac{e^{-y}}{-1}\right)dy\right\} = \frac{1}{2}\left[\frac{e^{-y}}{-1}\right]_0^\infty = \frac{1}{2} = 0.5$$

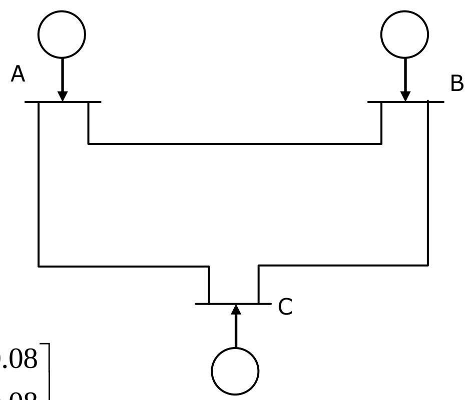
17. $$V'' = V\left(\frac{2Z_2}{Z_1 + Z_2}\right) = 100\left(\frac{2\times 50}{400+50}\right) = 22.22\,\text{kV}$$

18. 8 bit Johnson counter will divide frequency by 16 times
4 bit parallel counter will divide frequency by 16 times
8 bit ring counter will divide frequency by 8 times
Input frequency = 16 × 6 × 8 × 10 = 20480

19. $y = 0.02 + j0.08$

$y_{AB} = y_{BA} = y \quad y_{BC} = y_{CB} = y$

$y_{CA} = y_{AC} = y \quad y_{AA} = y + y = 2y$



$$\begin{bmatrix} y_{AA} & y_{AB} & y_{AC} \\ y_{BA} & y_{BB} & y_{BC} \\ y_{CA} & y_{CB} & y_{CC} \end{bmatrix} = \begin{bmatrix} 2y & -y & -y \\ -y & 2y & -y \\ -y & -y & 2y \end{bmatrix}$$

$$Y_{Bus} = \begin{bmatrix} 0.04 + j0.16 & -0.02 - j0.08 & -0.02 - j0.08 \\ -0.02 - j0.08 & 0.04 + j0.16 & -0.02 - j0.08 \\ -0.02 - j0.08 & -0.02 - j0.08 & 0.04 + j0.16 \end{bmatrix}$$

20. $dy = \left(\frac{2}{x} - 4x^2\right)dx \Rightarrow y = 2\ln x - \frac{4x^3}{3} + c$

21. To avoid phase displacement, the connection of power Transformer and CT’s as shown below:

PT’s		CT’s
If Δ / Y	$\rightarrow$	Y / Δ
If Δ / Δ	$\rightarrow$	Y / Y

22. Rotational loss = 500W

Field circuit loss = $200 \times 2w = 400w$

Constant loss = $(500 + 400)w = 900w$

For rated load

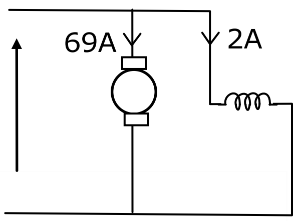
$E_b I_a = 10 \times (10^3)$

$\Rightarrow (200 - 0.8I_a)I_a = 10000 \Rightarrow I_a = 69A$

[$I_a = 181A$ is not feasible as it will result in huge losses]

So input current = 71 A = $71 \times 200w$ = 14.2kW

Efficiency = $\left(\frac{10}{14.2} \times 100\right)\%$ = 70.4%



23. $\tau_{dt} = \frac{P}{\omega_m} = \frac{895}{1710 \times \frac{2n}{60}} = 5\ N-m$

$\frac{\tau_{d_1}}{\tau_{d_2}} = \frac{V_{a_1}^{\ 2}}{V_{a_2}^{\ 2}} \Rightarrow \tau_{d_2} = \left(\frac{V_{a_2}}{V_{a_1}}\right)^2 \tau_{d_1} = (0.9)^2 \times 5 = 4.05\,Nm$

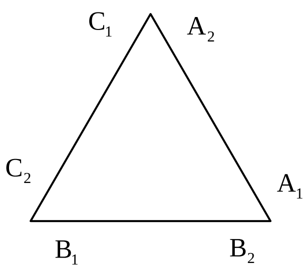
24. Upper cross over voltage when, v_o = +10v, At upper threshold point

$\left(\frac{5}{5+20}\right)10 = \left(\frac{10}{10+20}\right)2 + \left(\frac{20}{10+20}\right)V_{th} \Rightarrow V_{Th} = 2V$

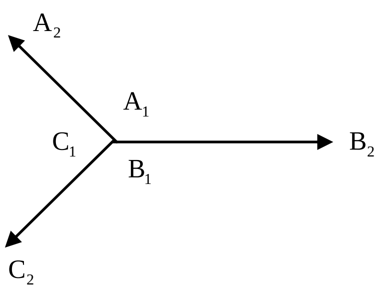
Lower cross over voltage when v_o = -10V

$\left(\frac{5}{5+20}\right)(-10) = \left(\frac{10}{10+20}\right)(2) + \left(\frac{20}{10+20}\right)V_{TL} \Rightarrow V_{TL}$ = -4V

25.



ΔY11



26. $V_1 = Av_2 + B(-I_2)$

$I_1 = CV_2 + D(-I_2)$

Making $V_2 = 0$

$I_1 = -I_2$

or $\left[D = \frac{I_1}{-I_2} = 1 \right]$

Making $I_2 = 0$

$V_2 = 4(I_1 - 0.1V_2)$

$V_2 = 4I_1 - 0.4V_2$

$1.4V_2 = 4I_1$

$V_1 - 5I_1 + 0.3V_1 = 0$

$1.3V_t = 5I_1 = -5I_2$

$\left[\frac{V_1}{-I_2} = B = \frac{5}{1.3} = 3.846 \right]$ or $\left[C = \frac{I_1}{V_2} = \frac{1.4}{4} = 0.35 \right]$

$V_1 - 5I_1 + 0.3V_1 - V_2 = 0$

$1.3V_1 - 5 \times 0.35V_2 - V_2 = 0$

$1.3V_1 = 2.75V_2$

or $\left[A = \frac{V_1}{V_2} = 2.11 \right]$ or $\begin{bmatrix} 2.11 & 3.846 \\ 0.33 & 1 \end{bmatrix}$

27. Find out the thevenin voltage across the galvanometer

$$E_{th} = \frac{E.R_3}{R_1 + R_3} - \frac{ER_4}{R_2 + R_4}; R_{th} = \frac{R_1R_3}{R_1 + R_3} + \frac{R_2R_4}{R_2 + R_1}$$

$$I_g = \frac{E_{th}}{R_{th} + R_g}$$

28. First multiplier output=10cos4000 πt

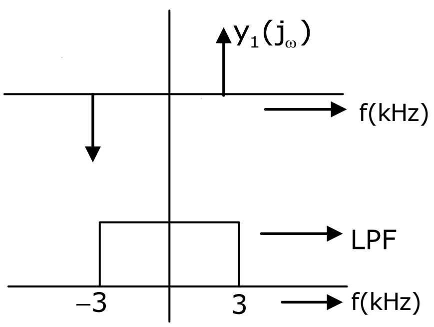
$\cos 4000\pi t$ is shifted by $90^o \Rightarrow \cos(4000\pi \pm 90)t = \sin 4000\pi t$

$\therefore$ second multiplier output $= 10\cos 4000\pi t \sin 4000\pi t$

$$y_1(t) = 5\sin 8000\pi t$$

$$y_1(i\omega) = \frac{5\pi}{j}\left[\delta(\omega - 8000\pi) - \delta(\omega + 8000\pi)\right]$$

$y_1(t)$ is passed through an LPF of cut off frequency 3kHz.



On multiplying above two signals the output y(t) is zero.

29. For the 1st stage alone,

$$A_V = \frac{-\beta_0 R_C}{R_S + r_\pi}; \quad r_\pi = \frac{\beta_0}{g_m} = 125 \times 25 = 3.125 k\Omega;$$

$$A_{V_1} = -\frac{125 \times 1.2}{0.6 + 3.125} = -40.3$$

For the 2nd stage, $R_S = R_{C_1}$

$= 1.2k$ & r_π is same $R_{C_2} = 1.2k$

$$\therefore A_{V_2} = \frac{-125 \times 1.2}{1.2 + 3.125} = -34.7$$

$\therefore$ Overall voltage gain $A_V = A_{V_1} \times A_{V_2} = 1400$

30. Assume H for r>b be Hex

$2\pi r H_{ex}$ = current enclose $= k.\pi(b^2 - a^2)$

$$H_{ex} = \frac{k\pi(b^2 - a^2)}{2\pi r}$$