**Answer Keys:**

**Section-I**

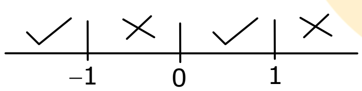
| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | B | **2** | D | **3** | C | **4** | B | **5** | A | **6** | A | **7** | D |
| **8** | A | **9** | D | **10** | C | **11** | D | **12** | B | **13** | D | **14** | D |
| **15** | D | **16** | D | **17** | A | **18** | A | **19** | D | **20** | B | | |

**Section-II**

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | A | **2** | D | **3** | A | **4** | B | **5** | A | **6** | A | **7** | A |
| **8** | D | **9** | C | **10** | C | **11** | C | **12** | A | **13** | A | **14** | B |
| **15** | D | **16** | B | **17** | D | **18** | A | **19** | A | **20** | A | **21** | A |
| **22** | A | **23** | C | **24** | B | **25** | A | **26** | B | **27** | B | **28** | D |
| **29** | D | **30** | C | | | | | | | | | | |

**Explanations:**

**Section-I**

1. 


For $x > 1$, $(x-1)x(x+1) \le 0$ condition is not satisfied

For $0 \le x \le 1$, $(x-1)x(x+1) \le 0$ condition is satisfied

$-1 < x < 0$, $(x-1)x(x+1) \le 0$ condition is not satisfied

$x \le -1$, $(x-1)x(x+1) \le 0$ condition is satisfied

2. Since 70% of the employees received bonuses of at least 10,000, 30% of the employees received bonuses of less than 10,000. We know that 60 employees received bonuses of less than 10,000. If E is the number of employees, we can set up the following equation:
.30E = 60, E = 200
40% of the employees received bonuses of at least 50,000. Thus, (40% × 200) or 80 employees received bonuses of at least 50,000. 20% of the employees received bonuses of at least 1,00,000. Thus, (20% × 200) or 40 employees received bonuses of at least 1,00,000. If 80 employees received at least 50,000, and 40 employees received at least 1,00,000, then (80 - 40) or 40 employees received bonuses of at least 50,000 but less than 1,00,000.

3. $P\left(1+\frac{r}{100}\right)^5 = 3P, \quad P\left(1+\frac{r}{100}\right)^{10} = 9P$

4. Average = sum of terms / number of terms. In this question, we can apply the formula to the difference between the average we got and the average we were supposed to get:

$1.8 = \frac{\text{extra sum of terms}}{10} \Rightarrow 18 = \text{extra sum of terms}$

So, 'ut' is 18 more than 'tu'

'ut'= $10u+1\times t=10u+t$

'tu'=$10t+1\times u=10t+u$

'ut' - 'tu' = 9 (u-t)

i.e. 18 = 9 (u-t)

Therefore, u-t=2

5. Let Tap A fills 1 liter in a minute.
Then Tap B fills 2 liters per min (that is why Tap A is taking double time)
Together, they will fill 3 lt per min.
In 6 hours, they will fill 18liters (which is capacity of tank).
To fill 18liters (full tank), Tap A will take 18 hours

6. When there is a loss at 10% $\rightarrow$ 160 = 90% of $CP_2$

$\therefore C.P_2 = 177.37$

When there is a profit of 10% $\rightarrow$ 160 = 110% of $CP_1$

$\therefore C.P_1 = 145.45$

Total $C.P = 177.77 + 145.45 = 323.23$

$\text{Loss} = 3.23$

7. Let the ages of children is $x, (x+3), (x+6), (x+9) \& (x+12)$ yrs.

Then $x + (x+3) + (x+6) + (x+9) + (x+12) = 50$

$5x + 30 = 50 \Rightarrow x = 4$

8. $x = \frac{90}{360} \times 45{,}000 = 11{,}250\,\text{rs}; \quad y = \frac{120}{360} \times 45{,}000 = 15{,}000\,\text{rs}$

$z = \frac{150}{360} \times 45{,}000 = 18{,}750\,\text{rs};$

Hence in 1997 the costs are:

$x = 11{,}250 \times 1.1 = \text{Rs. } 12375$

$y = 15{,}000 \times 1.3 = \text{Rs. } 19500$

$z = 18{,}750 \times 1.2 = \text{Rs. } 22500$

Total cost = 12375 + 19500 + 22500 = 54375

9. $1-9 \quad 9\times 1\text{digits} = 9$

$10-99 \quad 90\times 2\text{digits} = 180$

$100-999 \quad 900\times 3\text{digits} = 2700$

$\overline{2889}$

$2777^{th}$ digit is of a 3 digit number

$2889-2777=112=37\times 3+1$

From 999, 37 numbers behind is 962. Its second digit is required answer. So answer is 6.

10. i. $\frac{\text{B in 2011}}{\text{C in 2012}} = \frac{15}{55} = \frac{3}{11}$

ii. Average $= \frac{10+15+40+40}{4} = \frac{105}{4} = 26.25$

iii. Percentage increase in C $= \frac{30\text{-}15}{15}\times 100\% = 100\%$

## Section-II: Technical

1. The two balls drawn may be both green, one green and one red or both red.

In these cases, the man receives 40paise, 30paise and 20paise respectively.

Let X be the amount the man receives. Then

$P[x=40] = P[\text{both green}] = \frac{^3C_2}{^5C_2} = 0.3$

$P[x=30] = P[\text{one green one red}] = \frac{3C_1\times 2C_1}{5C_2} = 0.6$

$P[x=20] = P[\text{both red}] = \frac{2C_2}{5C_2} = 0.1$

$\therefore$ Probability distribution of x is

| x | 40 | 30 | 20 | | |
|---|---|---|---|---|---|
| $P(x)$ | 0.3 | 0.6 | 0.1 | | |

$E(x) = \Sigma x P(x) = 40\times 0.3 + 30\times 0.6 + 20\times 0.1 = 32\text{paise}$

2. After $1^{st}$ clock pulse = 1101

After $2^{nd}$ clock pulse = 1110

:

:

So on after $6^{th}$ clock pulse =1011

3. $V_o = V\sin\omega t$

$$\frac{dv_o}{dt} = V\omega\cos\omega t$$

$$\max\text{imum output} = V\omega = 10\times10^6$$

$$V = \frac{10\times10^6}{2\times\pi\times2\times10^6} = 0.8V$$

4. We will have

$$Y = \overline{\overline{\overline{\overline{X.Y}}.\overline{\overline{(1.0)}}+\overline{\overline{x.y}}\ .\overline{\overline{(1.0)}}}}$$

$$= \overline{\overline{(x.y)(1.0)}+\overline{(x.y).(1.0)}}$$

$$= \overline{\overline{(x.y.1.0).(x.y.1.0)}} = x.y.1.0 = 0$$

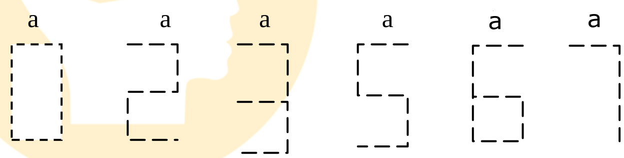
5. $\frac{\partial f}{\partial x} = nx^{n-1};\ \frac{\partial f}{\partial y} = ny^{n-1};\ \frac{\partial f}{\partial z} = nz^{n-1} \Rightarrow \nabla f = n\left[x^{n-1}i + y^{n-1}j + z^{n-1}k\right]$

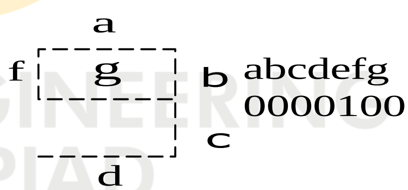
Since r = xi+yj+zk,

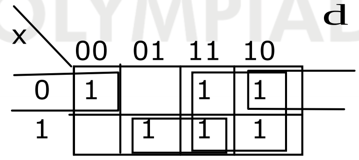
$\nabla f\cdot r = \left(nx^{n-1}x\right)+\left(ny^{n-1}y\right)+\left(nz^{n-1}z\right) = n\left(x^n+y^n+z^n\right) = nf$

7.

| x | y | z | a |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

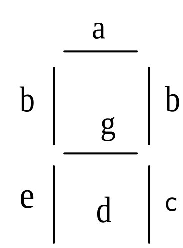



$a = y + xz + x'z'$

A seven-segment decoder with its segment display is shown below:
Segments outputs are given as a, b, c, d, e, f and g



8. $C = \frac{\varepsilon\theta r^2}{2d}$,

Sensitivity, $S = \frac{dC}{d\theta} = \frac{\varepsilon r^2}{2d}$ Independent of $\theta$

9. $8V = 1600I_D + V_{DS} + 400I_D,\ 8 = 2000I_D + V_{DS};\ 2000I_D = 8 - V_{DS}$

$I_D = \frac{8}{2000} - \frac{V_{DS}}{2000} = \left(\frac{-1}{2000}\right)V_{DS} + \left(\frac{8}{2000}\right),\ \boxed{m = \frac{-1}{2000}};$

$y = I_D;\ x = V_{DS}$

10. $C = C_P + C_C + C_A$

$= 1000 + 300(100) + 100 = 31100$ pF

R= 10.1 MΩ

$\tau = RC$ =0.314 sec

$\omega = 2\pi f = 2\pi(100) = 200\pi$ rad/sec

Phase shift $= \frac{\pi}{2} - \text{Tan}^{-1}(\tau\omega) = \frac{\pi}{2} - \left(\text{Tan}^{-1}\left(0.314 \times 200 \times \frac{22}{7}\right)\right) = 0.29^{o}$

11. $\frac{60R_{min}}{2000 + R_{min}} = 10 \Rightarrow R_{min} = 400\Omega$

$I_L = \frac{60 - 10}{2000} = 25\text{mA}$

$I_{L(min)} = 25\text{mA} - 20\text{mA} = 5\text{mA}$

$R_{L(max)} = \frac{10}{5m} = 2000\Omega$

$400\Omega < R < 2000\Omega$

12. $\underset{x\to\infty}{\text{Lt}}\left[\frac{x^2 + 5x + 3}{x^2 + x + 2}\right]^x = \underset{x\to\infty}{\text{Lt}}\left(1 + \frac{4x+1}{x^2+x+2}\right)^x = \underset{x\to\infty}{\text{Lt}}\left[\left(1 + \frac{4x+1}{x^2+x+2}\right)^{\frac{x^2+x+2}{4x+1}}\right]^{\frac{x(4x+1)}{x^2+x+2}}$

$= e^{\underset{x\to\infty}{\text{Lt}}\frac{x(4x+1)}{x^2+4x+2}} = e^{\underset{x\to\infty}{\text{Lt}}\frac{4+\frac{1}{x}}{1+\frac{1}{x}+\frac{1}{x^2}}} = e^4$

13. Given $R_P = 1000\Omega$

Resistance change due to temp= 1000 - 1Ω(90-50°) $= 960\Omega$

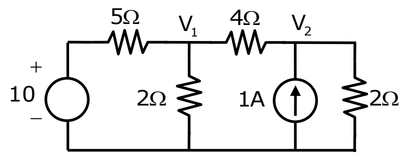
Max output voltage = 5×960×sin90°mV

= 4.8 volt

14. Applying KCL at $V_1$ and $V_2$

$$\frac{V_1-10}{5}+\frac{V_1}{2}+\frac{V_1-V_2}{4}=0 \qquad ....1$$

$$\frac{V_2-V_1}{4}+\frac{V_2}{2}=1 \qquad ....2$$




By solving (1) and (2), we will get

$V_1=2.7V, \quad V_2=2.23V$

15. $$z=\frac{10\angle 0}{5\angle -30}=2\angle 30; 10\cos(\omega t+30^o)=10\sin(90-\omega t-30^o)=10\sin(60-\omega t)$$

$$=-10\sin(\omega t-60)=-10\angle -60 V,$$

$$I=\frac{V}{Z}=\frac{-10\angle -60}{2\angle 30}=-5\angle -90=5\angle 90$$

$$=5\sin(\omega t+90)=5\cos\omega t \ A$$

16. $$\int_c Mdx+Ndy=\iint_R\left(\frac{\delta N}{\delta x}-\frac{\delta M}{\delta y}\right)dxdy, M=xy+y^2;\ N=x^2$$

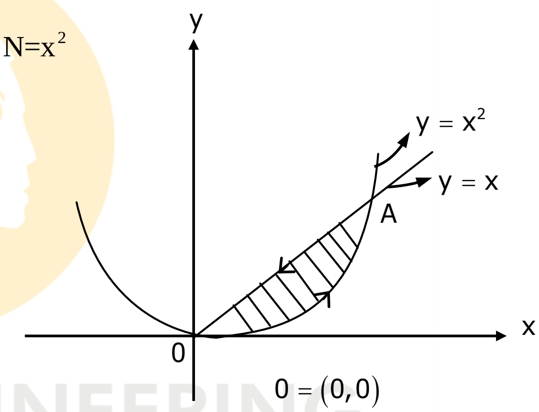
$$\frac{\delta N}{\delta x}-\frac{\delta M}{\delta y}=2x-(x+2y)=x-2y$$

$$\iint_R\left(\frac{\delta N}{\delta x}-\frac{\delta M}{\delta y}\right)dxdy$$

$$=\int_{x=0}^{1}\int_{y=x^2}^{x}(x-2y)dydx=\int_{x=0}^{1}\left[xy-y^2\right]_{y=x^2}^{x}dx$$

$$=\int_{x=0}^{1}\left[\left(x^2-x^2\right)-\left(x^3-x^4\right)\right]dx=\int_{x=0}^{1}\left(x^4-x^3\right)dx$$

$$=\frac{1}{5}-\frac{1}{4}=\frac{-1}{20}$$




$0=(0,0)$
$A=(1,1)$

y varies $x^2$ to x
x varies 0 to 1

17. Speed of shaft, $N=\dfrac{f_m f_1(m-1)}{f_m-f_1}$

$f_m$ – Max. RPM
$f_1$ – Min. RPM
m – No. of images

$$n=\frac{5000\times 2000(4-1)}{5000-2000}$$
$$=10000 \text{ rpm}$$

18. The closest scale marking w.r.t to the measured length is chosen. Here it is 231 cm (L).
Now with absolute certainty we can say that $L \pm 0.1$ cm. since 0.1 cm is the smallest measurement quantity in this scale.

19. $$e_o = \frac{Q}{C} = \frac{dF}{C} = \frac{d.ma}{C} = \frac{2\times10^{-12}\times0.01\times20}{100\times10^{-12}} = 4\,mV$$

20. We have, $e^{1/z} = 1 + \frac{1}{z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} .....$

$$\sin\frac{1}{z} = \frac{1}{z} - \frac{1}{3!z^3} + .....$$

There is an essential singularity at z = 0

The residue at z = 0 is coefficient of $\frac{1}{z}$ in Laurent series of integrand, which is 1

So $\oint_{|z|=1} e^{1/z} \sin\frac{1}{z} dz = 2\pi i$

21. Given $\cos\phi = 0.3 \Rightarrow \phi = 1.266$

$R_p = 2500\Omega;\ L = 20\,mH;\ V = 120V;\ I = 10A$

Power consumed by load, $P_T = VI\cos\phi = 120\times10\times0.3 = 360W$

$$\beta = \tan^{-1}\left(\frac{X_L}{R_p}\right) = \tan^{-1}\left(\frac{2\pi fL}{R_p}\right)$$

$$= \tan^{-1}\left(\frac{2\pi\times50\times20\times10^{-3}}{2500}\right) = 0.00251 \text{ rad}$$

Actual wattmeter reading = $= [1 + \tan\phi\ \tan\beta]$ true power

$$= [1 + \tan(72.54°)\ \tan(0.143°)]360 = 362.87\,w$$

Power loss, $P_L = \frac{V^2}{R_p} = \frac{(120)^2}{2500} = 5.76W$

Total wattmeter reading = $362.84 + 5.76 = 368.63\,W$

$$\therefore \%\text{Error} = \frac{P_w - P_T}{P_T}\times100 = \frac{368.63 - 360}{360}\times100 = 2.397\%$$

22. $\text{Sensitivity} = \frac{\Delta E}{x}$

$$10\ V/m = \frac{\Delta E}{0.1} \Rightarrow \Delta E = 1V$$

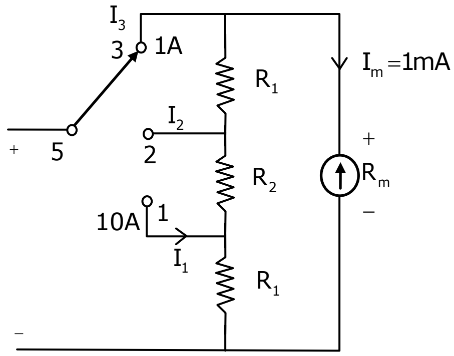
23. In position 1, $R_1$ is in shunt with $R_2 + R_3 + R_m$

$\therefore I_1R_1 = I_m(R_2 + R_2 + R_3)$

Given $I_1 = 10A$

$I_m = 1mA$

$R_m = 50\Omega$




$\therefore 10R_1 = 1\times10^{-3}[R_2 + R_3 + 50]$

$\Rightarrow R_1 = 10^{-4}[R_2 + R_3 + 50]$

In position 2, $R_1 + R_2$ is in shunt with $R_3 + R_m$

$I_2(R_1 + R_2) = I_m(R_3 + R_m)$

$\uparrow$

5A $5(R_1 + R_2) = I_m(R_3 + R_m)$

$5(R_1 + R_2) = 1\times10^{-3}[R_3 + 50]$

$\therefore R_1 + R_2 = 1\times10^{-3}[R_3 + 50] \Rightarrow R_1 + R_2 = 2\times10^{-4}[R_3 + 50]$

In position 3, $R_1 + R_2 + R_3$ is in shunt with $R_m$

$I_3(R_1 + R_2 + R_3) = I_mR_m$

$\downarrow$

1A $\therefore R_1 + R_2 + R_3 = 1\times10^{-3}\times50 = 0.05$

$R_1 + R_2 + R_3 = 0.05$

$R_1 + R_2 = 2\times10^{-4}[R_3 + 50] \Rightarrow R_3 = 0.0399\Omega$

$R_1 + R_2 = 0.01, \quad R_2 = 0.01 - R_1$

$R_1 = 10^{-4}(R_2 + R_3 + 50)$

$R_1 = 10^{-4}[0.01 - R_1 + 0.0399 + 50]$

$\therefore R_1 = 0.005\Omega, \quad R_2 = 0.005\Omega$

24. $L_{eq} = 4 + 8 + 6 - (2\times2) + (2\times4) = 22H$

25. $\omega = 0, \; V_{out} = V_{in}$

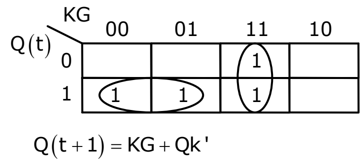
$\omega = \infty, \; V_{out} = 0$

It represents low pass filter

26. $A_v = \dfrac{-h_{fe}R_c}{h_{ie} + (1 + h_{fe})R_e} = -20$

27.

| Q(t) | K | G | Q(t+1) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |




$Q(t+1) = KG + Qk'$

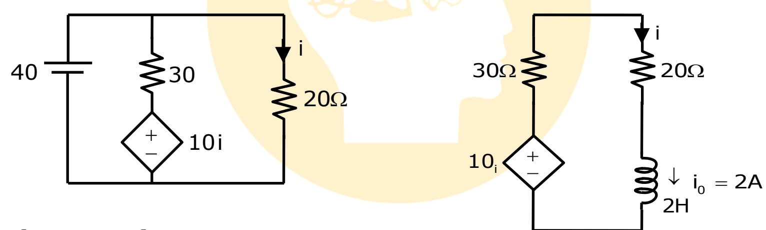
28. Given $R_1 = 4.5K\Omega;\ \ C_1 = 1\mu F$

$R_2 = 6.5K\Omega;\ \ R_3 = 650\Omega$

$\omega = 1000 \text{ rad} \times \sec$

$$R_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{1+\omega^2 R_1^2 C_1^2} = \frac{(1000)^2 \times (4.5\times10^3)\times(1\times10^{-6})^2 \times (6.5\times10^3)\times(650)}{1+(1000)^2(4.5\times10^3)(1\times10^{-6})^2} = 894.7\Omega$$

29. $t = 0^- \qquad \therefore i(0) = \frac{40}{20} = 2A$




at $t = 0^+$ the circuit becomes

$$10\,i(t) = 30i(t) + 20i(t) + 2\frac{di}{dt}$$

$$2\frac{di}{dt} + 40\,i(t) = 0$$

$$i(t) = k\,e^{-20t}$$

$$i(t) = 2e^{-20t} \qquad i(0) = 2A$$



30. $V_0 = -\left(\frac{R}{R}\right) - \left(\frac{R}{R}\right)V_1 = V_1$